Year 11 into 6th form Further Mathematics A level induction

Expectations

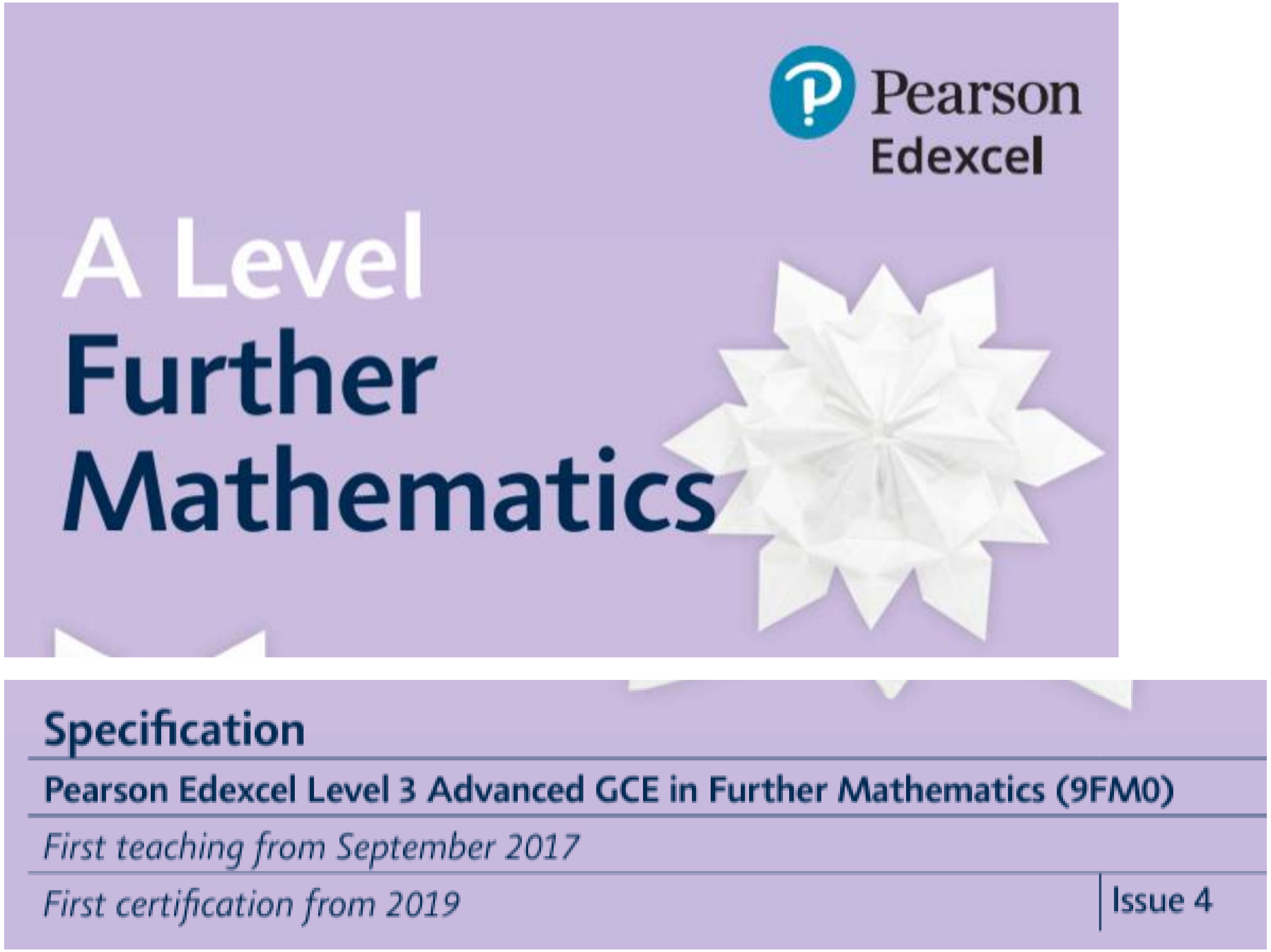
- Attending lessons
- Notes from lessons
- Exercises completed at home
- A file / folder to organise your work
- Independent exercises – every two weeks
- In class tests
- End of year 12 AS Exam (3 papers)
- No IPEs in Y12 or Y13
- End of year 13 GCE exam (4 papers)

Entry requirements

- Taking A level mathematics

- Grade 8 at GCSE
- Dr Frost (Preparation) summer work

The course specification



Pearson Edexcel

A Level
Further
Mathematics

Specification

Pearson Edexcel Level 3 Advanced GCE in Further Mathematics (9FM0)

First teaching from September 2017

First certification from 2019 | Issue 4

Assessment: Summer 2025

Qualification	Component	Overview	Assessment
AS level Further Mathematics	**Paper 1: Core Pure Mathematics** 50% weighting	Compulsory content – AS level Pure Mathematics	1 hour 40 mins 80 marks
	Paper 2: Further Mathematics Options 50% weighting	Students take two options assessed in one paper. Options available in: • Further Pure Mathematics 1 and 2 s 1 cs 1 matics See below for details of how these options can be arranged	1 hour 40 mins 80 marks

Further Mechanics 1

Decision Mathematics 1

Assessment: Summer 2026

Qualification	Component	Overview	Assessment
A level Further Mathematics	**Paper 1: Core Pure Mathematics 1** 25% weighting	Compulsory content – any Pure Mathematics content can be assessed on either paper	1.5 hours 75 marks
	Paper 2: Core Pure Mathematics 2 25% weighting		1.5 hours 75 marks
	Paper 3: Further Mechanics 1 25% weighting	Students take two … with … in: … Mathematics 1 and 2 • Further Statistics 1 and 2 • Further Mechanics 1 … ematics 1 and 2 See below for details of how these options can be arranged	1.5 hours 75 marks
	Paper 4: Decision Mathematics 1 25% weighting		1.5 hours 75 marks

Assessment: Content

Pure Mathematics:
Proof
Complex numbers
Matrices
Further algebra and functions
Further Calculus
Further vectors
Polar coordinates
Hyperbolic functions
Differential Equations

Further Mechanics 1
Momentum and impulse
Work energy and power
Elastic strings and springs and elastic energy
Elastic collisions in one dimension
Elastic collisions in two dimensions

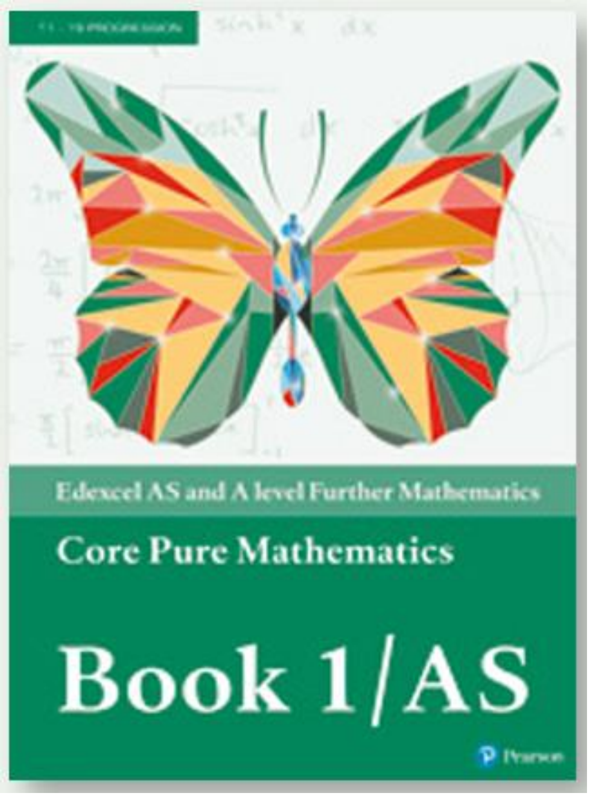
Decision mathematics 1
Algorithms and graph theory
Algorithms on graphs
Critical path analysis
Linear programming

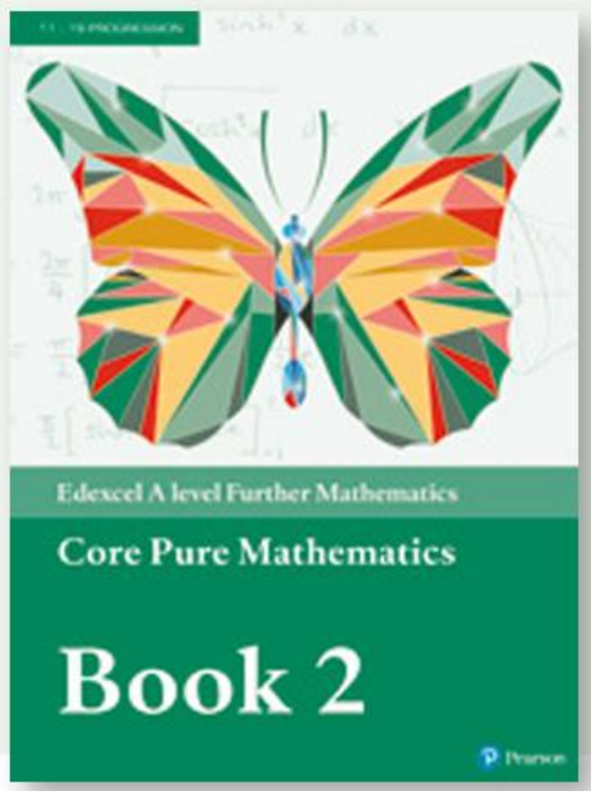
Calculators

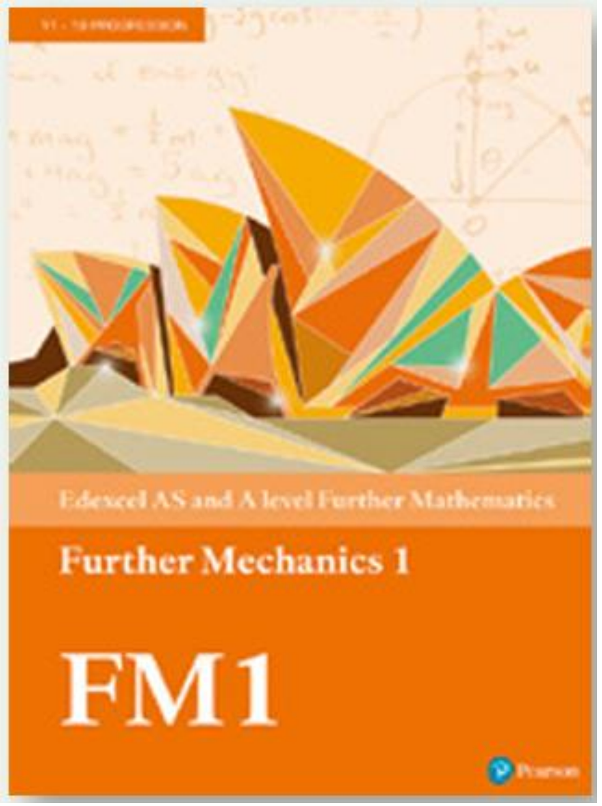
- Casio fx-991EX
  You must buy this essential tool, necessary for Statistics calculations. Approximate cost is around £30

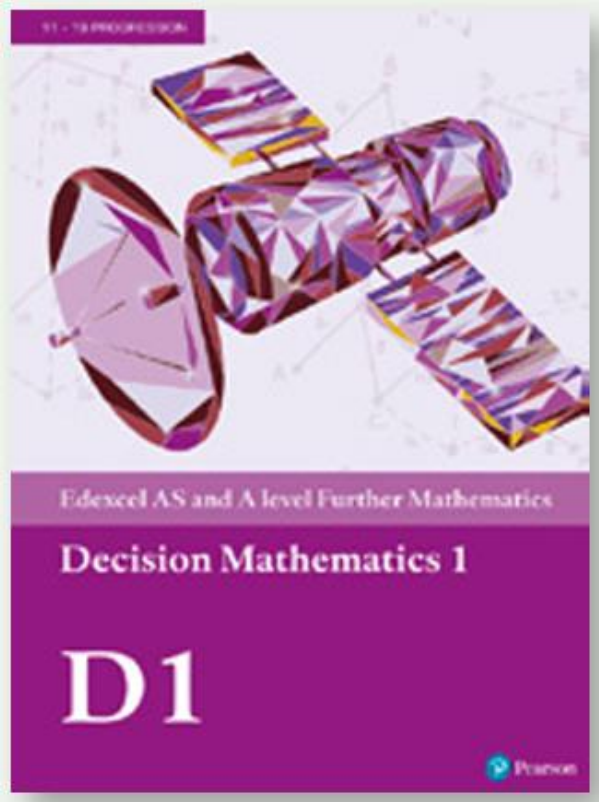
- Casio fx-cg50
  An optional graphical calculator. Approximate cost is around £120 but if purchased through school Casio discounts this to around £90.

The textbook is provided in e-book form









Purpose of induction activity

- A reminder that GCSE knowledge and skills will be assumed and necessary for the A level course.
- An introduction to the style and standard of the easiest questions.
- An introduction to how exam questions are assessed.

- The questions that follow whilst being AS questions are fully based on knowledge gained for the GCSE examination.

Practice questions: method is essential

The Edexcel Mathematics mark schemes use the following types of marks:

M marks: method marks are awarded for ‘knowing a method and attempting to apply it’, unless otherwise indicated.

A marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.

B marks are unconditional accuracy marks (independent of M marks)

24 marks ≈ 24 minutes

Why study complex numbers?

- All numbers are imaginary (even "zero" was contentious once). Introducing the square root(s) of minus one is convenient because (i) all n-degree polynomials with real coefficients then have n roots, making algebra "complete";

- They are of enormous use in applied maths and physics. Complex numbers (the sum of real and imaginary numbers) occur quite naturally in the study of quantum physics. They're useful for modelling periodic motions (such as water or light waves) as well as alternating currents. Understanding complex analysis, the study of functions of complex variables, has enabled mathematicians to solve fluid dynamic problems particularly for largely 2 dimensional problems where viscous effects are small. You can also understand their instability and progress to turbulence. All of the above are relevant in the real world, as they give insight into how to pump oil in oilrigs, how earthquakes shake buildings and how electronic devices (such as transistors and microchips) work on a quantum level (increasingly important as the devices shrink.)

https://www.theguardian.com/notesandqueries/query/0,5753,-18864,00.html#:~:text=They%20are%20of%20enormous%20use,as%20well%20as%20alternating%20currents.

Introduction

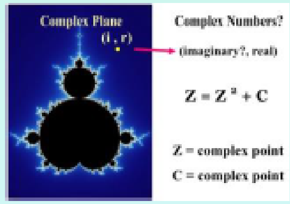
- The first chapter of FP1 introduces you to imaginary and complex numbers

- You will have seen at GCSE level that some quadratic equations cannot be solved

- Imaginary and complex numbers will allow us to actually solve these equations!

- We will also see how to represent them on an Argand diagram

- We will also see how to use complex numbers to solve cubic and quartic equations



Complex Numbers

You can use both real and imaginary numbers to solve equations

At GCSE level you met the Quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The part under the square root sign is known as the 'discriminant', and can be used to determine how many solutions the equation has:

$$b^2 - 4ac > 0 \quad \dashrightarrow 2 \, real \, roots$$

$$b^2 - 4ac = 0 \quad \dashrightarrow 1 \, real \, root$$

$$b^2 - 4ac < 0 \quad \dashrightarrow 0 \, real \, roots$$

The problem is that we cannot square root a negative number, hence the lack of real roots in the 3rd case above

To solve these equations, we can use the imaginary number 'i'

$$i = \sqrt{-1}$$

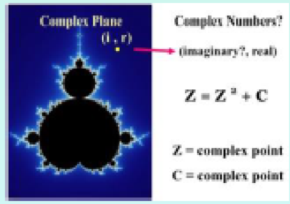
The imaginary number 'i' can be combined with real numbers to create 'complex numbers'

An example of a complex number would be:

$$5 + 2i$$

Complex numbers can be added, subtracted, multiplied and divided in the same way you would with an algebraic expression

1A



Complex Numbers

You can use both real and imaginary numbers to solve equations

1) Write √-36 in terms of i

This sign means the **positive** square root →

$\sqrt{-36}$

$\sqrt{36}\sqrt{-1}$

$= 6i$

Split up using surd manipulation

Simplify each part → √-1 = i

2) Write √-28 in terms of i

$\sqrt{-28}$

$\sqrt{28}\sqrt{-1}$

$\sqrt{4}\sqrt{7}\sqrt{-1}$

$= 2\sqrt{7}i$

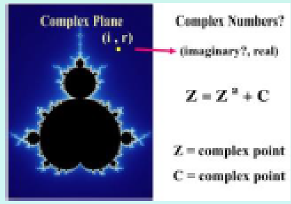
$= 2i\sqrt{7}$

Split up into a positive and negative part

Split up the 28 further…

Simplify each part

This is usually written in this way



Complex Numbers

You can use both real and imaginary numbers to solve equations

Solve the equation:

$$x^2 + 9 = 0$$

$$x^2 + 9 = 0$$

Subtract 9

$$x^2 = -9$$

Square root – we need to consider both positive and negative as we are solving an equation

$$x = \pm\sqrt{-9}$$

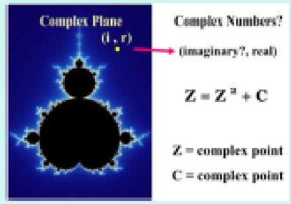
Split up

$$x = \pm\sqrt{9}\sqrt{-1}$$

Write in terms of i

$$x = \pm 3i$$

You should ensure you write full workings – once you have had a lot of practice you can do more in your head!



Complex Numbers

You can use both real and imaginary numbers to solve equations

Solve the equation:

$$x^2 + 6x + 25 = 0$$

→ You can use one of two methods for this

→ Either 'Completing the square' or the Quadratic formula

$(x+3)^2$

Imagine squaring the bracket

$(x+3)(x+3)$

This is the answer we get

$x^2 + 6x + 9$

The squared bracket gives us both the x^2 term and the 6x term

→ It only gives us a number of 9, whereas we need 25 – add 16 on!

Completing the square

$x^2 + 6x + 25 = 0$

Write a squared bracket, with the number inside being half the x-coefficient

$(x+3)^2 + 16 = 0$

Subtract 16

$(x+3)^2 = -16$

Square root

$x + 3 = \pm\sqrt{-16}$

Subtract 3

$x = -3 \pm \sqrt{-16}$

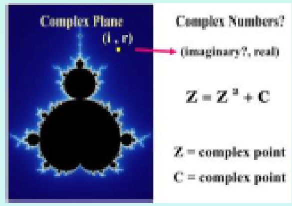
Split the root up

$x = -3 \pm \sqrt{16}\sqrt{-1}$

Simplify

$x = -3 \pm 4i$

If the x term is even, and there is only a single x^2, then completing the square will probably be the quickest method!



Complex Numbers

You can use both real and imaginary numbers to solve equations

Solve the equation:

$$x^2 + 6x + 25 = 0$$

→ You can use one of two methods for this

→ Either 'Completing the square' or the Quadratic formula

a = 1

b = 6

c = 25

The Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Sub in values

$$x = \frac{-6 \pm \sqrt{(6)^2 - (4 \times 1 \times 25)}}{2(1)}$$

Calculate the part under the root sign

$$x = \frac{-6 \pm \sqrt{-64}}{2}$$

Split it up

$$x = \frac{-6 \pm \sqrt{64}\sqrt{-1}}{2}$$

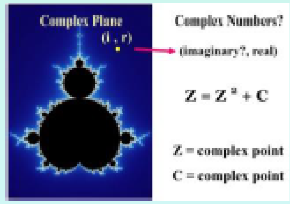
Simplify the roots

$$x = \frac{-6 \pm 8i}{2}$$

Divide all by 2

$$x = -3 \pm 4i$$

If the x^2 coefficient is greater than 1, or the x term is odd, the Quadratic formula will probably be the easiest method!



Complex Numbers

You can use both real and imaginary numbers to solve equations

Simplify each of the following, giving your answers in the form:

$$a + bi$$

where:

$$a \in R \text{ and } b \in R$$

↑ This means a and b are real numbers

1) $(2 + 5i) + (7 + 3i)$

$= 9 + 8i$

Group terms together

2) $(2 - 5i) - (5 - 11i)$

$= 2 - 5i - 5 + 11i$

$= -3 + 6i$

'Multiply out' the bracket

Group terms

3) $6(1 + 3i)$

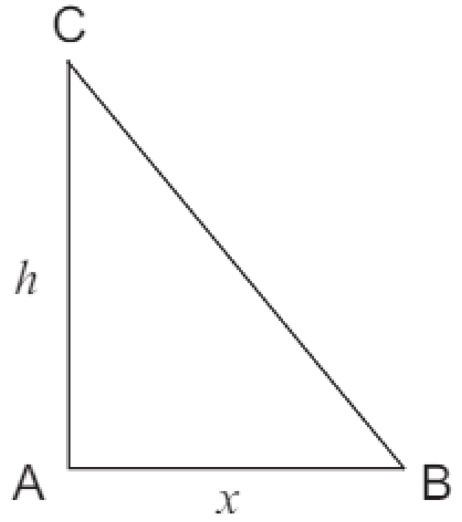
$= 6 + 18i$

Multiply out the bracket

Questions?

For example, *Diophantus* (about 275 AD) attempted to solve what seems a reasonable problem, namely

> 'Find the sides of a right-angled triangle of perimeter 12 units and area 7 squared units.'



Letting $AB = x$, $AC = h$ as shown,

then area $= \frac{1}{2} x h$

and perimeter $= x + h + \sqrt{x^2 + h^2}$

Activity 1

Show that the two equations above reduce to

$$6x^2 - 43x + 84 = 0$$

when perimeter $= 12$ and area $= 7$. Does this have real solutions?

A similar problem was posed by *Cardan* in 1545. He tried to solve the problem of finding two numbers, a and b, whose sum is 10 and whose product is 40;

i.e. $$a + b = 10 \qquad (1)$$

$$ab = 40 \qquad (2)$$

Eliminating b gives

$$a(10 - a) = 40$$

or $$a^2 - 10a + 40 = 0\,.$$

Solving this quadratic gives

$$a = \frac{1}{2}(10 \pm \sqrt{-60}) = 5 \pm \sqrt{-15}$$

Activity 2 The need for complex numbers

Solve if possible, the following quadratic equations by factorising or by using the quadratic formula. If a solution is not possible explain why.

(a) $x^2 - 1 = 0$ (b) $x^2 - x - 6 = 0$

(c) $x^2 - 2x - 2 = 0$ (d) $x^2 - 2x + 2 = 0$

Activity 3

Solve the following equations, leaving your answers in terms of i:

(a) $x^2 + x + 1 = 0$ (b) $3x^2 - 4x + 2 = 0$

(c) $x^2 + 1 = 0$ (d) $2x - 7 = 4x^2$

Addition and subtraction

Addition of complex numbers is defined by separately adding real and imaginary parts; so if

$$z = a + bi,\ w = c + di$$

then $$z + w = (a + c) + (b + d)i\,.$$

Similarly for **subtraction**.

Example

Express each of the following in the form $x + yi$.

(a) $(3 + 5i) + (2 - 3i)$

(b) $(3 + 5i) + 6$

(c) $7i - (4 + 5i)$

Solution

(a) $(3 + 5i) + (2 - 3i) = 3 + 2 + (5 - 3)i = 5 + 2i$

(b) $(3 + 5i) + 6 = 9 + 5i$

(c) $7i - (4 + 5i) = 7i - 4 - 5i = -4 + 2i$

Multiplication is straightforward provided you remember that $i^2 = -1$.

Example

Simplify in the form $x + yi$:

(a) $3(2+4i)$

(b) $(5+3i)i$

(c) $(2-7i)(3+4i)$

Solution

(a) $3(2+4i) = 3(2)+3(4i) = 6+12i$

(b) $(5+3i)\,i = (5)\,i+(3i)\,i = 5i+3\left(i^2\right) = 5i+(-1)3 = -3+5i$

(c) $(2-7i)(3+4i) = (2)(3) - (7i)(3) + (2)(4i) - (7i)(4i)$

$$= 6-21i+8i-(-28)$$

$$= 6-21i+8i+28$$

$$= 34-13i$$

Activity 5

Simplify the following expressions:

(a) $(2+6i)+(9-2i)$

(b) $(8-3i)-(1+5i)$

(c) $3(7-3i)+i(2+2i)$

(d) $(3+5i)(1-4i)$

(e) $(5+12i)(6+7i)$

(f) $(2+i)^2$

(g) i^3

(h) i^4

(i) $(1-i)^3$

(j) $(1+i)^2+(1-i)^2$

(k) $(2+i)^4+(2-i)^4$

(l) $(a+ib)(a-ib)$